Wang Laboratories, Inc.

LOCI User Application Reply #1

We thank you for your prompt reply, giving us your full address. The details of the application are enclosed as you requested.

Please remember that if you have any applications of general interest, we shall be happy to distribute it.

Sincerely,

WANG LABORATORIES, INC.

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NC/bw
Enc. L29-66

LOCI Engineering Application # 5

(Curve Fitting and Interpolation)

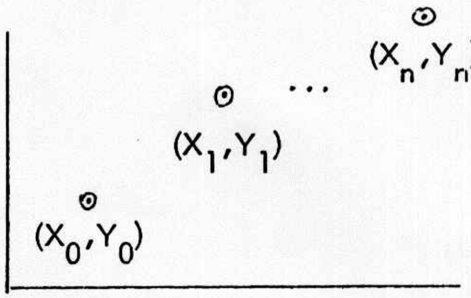
Lagrangian Interpolation

Unequal Spacing

A problem which often occurs in engineering analysis and statistics is to interpolate between known data points. A simple classical technique for doing this is the method of Lagrangian Interpolation. Suppose we use n points taken from observations:

$$(X_0, Y_0), (X_1, Y_1), \ldots (X_n, Y_n).$$

The method gives an (n−1)-th degree polynomial approximation p (x) to the points in-between. The polynomial is given as follows:



$$p(x) = a_0(X-X_1)(X-X_2)\ldots(X-X_n) + a_1(X-X_2)\ldots(X-X_n) + \ldots + a_n(X-X_0)(X-X_1)\ldots(X-X_{n-1}),$$

$$a_0 = \frac{Y_0}{(X_0-X_1)(X_0-X_2)\ldots(X_0-X_n)},$$

$$a_1 = \frac{Y_1}{(X_1-X_0)(X_1-X_2)\ldots(X_1-X_n)}$$

$$\vdots$$

$$a_n = \frac{}{(X_n-X_0)(X_n-X_1)\ldots(X_n-X_{n-1})}.$$

The interpolation is generally excellent for points that fall within the intervals between the known points.

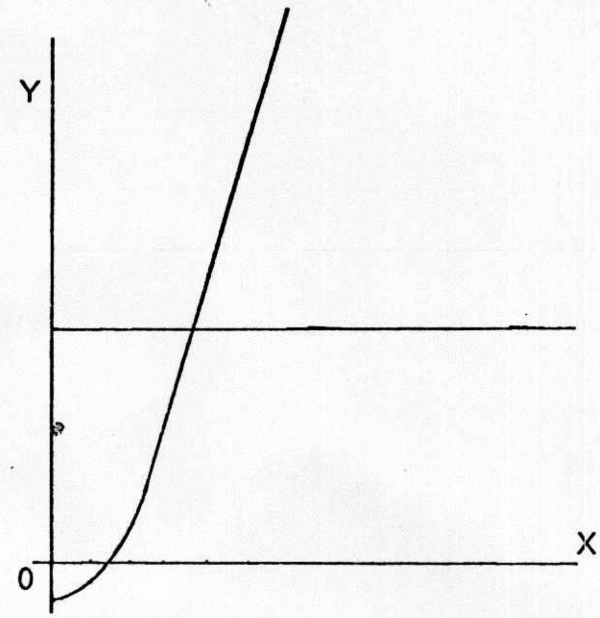
The LOCI program S123 uses seven points for a 6^{th} degree polynomial interpolation. Consider the following example, where we are given seven points.

Given:

X	Y
1	-1.75
2	-1
4	2
5	4.25
6	7
10	23
11	28.25

Want:

X	Y
3	?
4.5	?
7	?
8	?
9	?



The first card in program S123 is used to calculate the coefficients $a_0, \ldots, a_6$. Once this is done, the second card is used to make any number of calculations, for example, to interpolate at the points X = 3, 4.5, 7, 8, and 9.

Operation Instructions

Card #1 To operate the first card, key in a Y value, follow it with the corresponding X value, then follow that with all of the other X-values. Do this for all seven of the Y's. The coefficient values a_0-a_6 are automatically stored properly.

1. AUTO DISP down, [PRIME], card in reader
2. [1] [.] [7] [5] [±] [Po] (Yo)
3. [1] [RUN] (Xo)
4. [2] [RUN] (X1)
5. [4] [RUN] (X2)
6. [5] [RUN] (X3)
7. [6] [RUN] (X4)
8. [1] [0] [RUN] (X5)
9. [1] [1] [RUN] (X6)

$$\text{for } a_0 = \frac{Y_0}{(X_0-X_1)(X_0-X_2)\ldots(X_0-X_6)}$$

(-.00032407)

10. [1] [±] [RUN] (Y1)
11. [2] [RUN] (X1)
12. [1] [RUN] (X0)
13. [4] [RUN] (X2)
14. [5] [RUN] (X3)
15. [6] [RUN] (X4)
16. [1] [0] [RUN] (X5)
17. [1] [1] [RUN] (X6)

$$\text{for } a_1 = \frac{Y_1}{(X_1-X_0)(X_1-X_2)\ldots(X_1-X_6)}$$

(.00057870)

18. [2] [RUN] (Y2)
19. [4] [RUN] (X2)
20. [1] [RUN] (X0)
21. [2] [RUN] (X1)
22. [5] [RUN] (X3)
23. [6] [RUN] (X4)
24. [1] [0] [RUN] (X5)
25. [1] [1] [RUN] (X6)

$$\text{for } a_2 = \frac{Y_2}{(X_2-X_0)(X_2-X_1)(X_2-X_3)\ldots(X_2-X_6)}$$

.

.

.

Repeat for all Y's.

LOCI Program Lagrangian Interpolation Coefficients (Card # 1) Date: July 28, 1966

	0	4	8	12
S0		a_2	a_4	a_6
S1	X_i	X_2	X_4	X_6
S2	a_0	a_1	a_3	a_5
S3	X_0	X_1	X_3	X_5

$$a_0 = \frac{Y_0}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_6)}$$

$$a_1 = \frac{Y_1}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_6)}$$

$$\vdots$$

$$a_6 = \frac{Y_6}{(x_6-x_0)(x_6-x_1)\ldots(x_6-x_5)}$$

(1) PRIME, AUTO DISP down
(2) Key in Y_0, Po,
(3) Key in X_0, RUN,
(4) Key in X_1, RUN,
(5) Repeat (4) for all X_i
(6) Key in Y_1 RUN,
(7) Key in X_1, RUN,
(8) Key in X_0, RUN,
(9) Key in X_2, RUN.
(10) Repeat (9) for all X_i.
Repeat (6) - (10) for all remaining X-Y combinations.

No.	Cmd	Code	Comment	No.	Cmd	Code	Comment	No.	Cmd	Code	Comment	No.	Cmd	Code	Comment
00	X	12	Y_0	20	A→S0	50	a_4	40	÷	17		60			
1	3	23	23	21	MS	10		41	Dec	66		61			
2	2	22	22	22	Restore	65		42	DC=0	70		62			
3	Store	64		23	W→S2	54	a_5	43	3	23		63			
4	W→S2	54	a_0	24	S1→W	53		44	6	26		64			
05	S1→W	53		25	W→S3	56	x_3	45	W→PC	40		65			
6	W→S3	56	x_0	26	Restore	65		46	Ln^{-1}	14		66			
7	MS	10		27	A→S0	50	a_6	47	W→A	44		67			
8	Restore	65		28	MS	10		48	Restore	65		68			
9	W→S2	54	a_1	29	Stop	37		49	3	23	23	69			
10	S1→W	53		30	Stop	37	y_i	50	W→PC	40		70			
11	W→S3	56	x_1	31	X	12		51				71			
12	Restore	65		32	Stop	37	x_i	52				72			
13	A→S0	50	a_2	33	W→S1	52		53				73			
14	MS	10		34	6	26		54				74			
15	Restore	65		35	W→DC	42		55				75			
16	W→S2	54	a_3	36	S1→W	53	x_i	56				76			
17	S1→W	53		37	W→A	44		57				77			
18	W→S3	56	x_3	38	Stop	37	x_j	58				78			
19	Restore	65		39	-	15	(x_i-x_j)	59				79			

S123 - 1

Card # 2 The second card performs the interpolation. For our example:

1. [PRIME] [3] [Po] (Interpolate for X = 3.)
2. Read answer: .2499999266
3. [PRIME] [4] [·] [5] [Po] (Interpolate for X = 4.5)
4. Read answer: 3.062499965
5. [PRIME] [7] [Po] (Interpolate for X = 7)
6. Read answer: 10.24999986
7. [PRIME] [8] [Po] (Interpolate for X = 8)
8. Read answer: 14.00000063
9. [PRIME] [9] [Po] (Interpolate for X = 9)
10. Read answer: 18.24999921

Notes: The known data were, in fact, points on the curve

$$Y = 1/4X^2 - 2.$$

The quality of the above interpolations is evident.

Note that because of the programming procedure used, the points corresponding to $X_o, X_1, X_2, \ldots, X_6$ cannot be used.

LOCI Program Lagrangian Interpolation (7pt) (Card #2) **Date:** July 28, 1966

	0	4	8	12
S0	Σ	a_2	a_4	a_6
S1	X	X_2	X_4	X_6
S2	a_0	a_1	a_3	a_5
S3	X_0	X_1	X_3	X_5

$$p(x) = \left(\frac{a_6}{x-x_6} + \frac{a_0}{x-x_0} + \frac{a_1}{x-x_1} + \ldots + \frac{a_5}{x-x_5}\right)\Pi$$

$$= \left(\Sigma \frac{a_i}{x-x_i}\right)\Pi$$

$$\Pi = (x-x_6)(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)$$

$$x_0 \neq x_1 \neq x_2 \neq x_3 \neq x_4 \neq x_5 \neq x_6$$

To find p(x): AUTO DISP down

(1) Prime
(2) Key in X
(3) Po

Program no good for

$x = x_0, x_1, x_2, \ldots, x_6$

No.	Cmd	Code	Comment	No.	Cmd	Code	Comment	No.	Cmd	Code	Comment	No.	Cmd	Code	Comment
00	A→S0	50		20	CLW	02		40	S1→W	53	X	60	W→PC	40	
1	W→A	44	X	21	DC=0	70		41	W→A	44		61	S3→W	57	
2	W→S1	52		22	6	26	1st time	42	MS	10		62	—	15	X - Xi
3	2	22		23	1	21		43	Restore	65		63	X	12	Π (X - Xi)
4	W→DC	42		24	P2	62	Go find Π	44	S1→W	53	$X_{0,2,4}$	64	CLW	02	
05	3	23		25	S0→A	51	2nd time	45	—	15		65	S1→W	53	X
6	Store	64	X_6	26	A→W	45		46	÷	17		66	W→A	44	
7	Restore	65	X_0	27	X	12		47	S0→A	51	$a_{0,2,4}$	67	MS	10	
8	Restore	65	X_1	28	Ln^{-1}	14		48	A→W	45		68	Restore	65	
9	MS	10		29	Stop	37		49	X	12	$a_i/(X - Xi)$	69	S1→W	53	
10	Restore	65	X_2	30	S3→W	57		50	CLW	02		70	—	15	X - Xi
11	MS	10		31	-	15	$X - X_{6,1,3,5}$	51	S0→A	51	Σ	71	X	12	Π (X - Xi)
12	Restore	65	X_3	32	÷	17		52	Ln^{-1}	14		72	CLW	02	
13	MS	10		33	S2→W	55	$a_{6,1,3,5}$	53	+	13		73	S1→W	53	
14	MS	10		34	X	12		54	A→S0	50		74	W→A	44	
15	Restore	65	X_4	35	CLW	02		55	S1→W	53	X	75	MS	10	
16	MS	10		36	S0→A	51	$a_i/(X-Xi)$	56	W→A	44		76	Restore	65	
17	MS	10		37	Ln^{-1}	14		57	MS	10		77	6	26	
18	Restore	65	X_5	38	+	13	Σ	58	Restore	65		78	1	21	
19	DEC	66		39	A→S0	50		59	3	23		79	W→PC	40	