



LABORATORIES, INC. 836 NORTH ST., TEWKSBURY, MASS. 01876, TEL. (617) 851-7311

Calculating & Computing Applications

February 24, 1967

## LOCI User Application Letter # 1 — Introduction

We are initiating a LOCI User Application letter for the purpose of circulating programs and applications which may be of common interest amongst all of our LOCI users. With over 300 LOCI's in the field, and all being used continuously, we feel that there are many users who have developed extremely useful programs which are of general value. While many programs related to the specific applications of the user cannot be released due to general company policy, there may be many programs which are releasable.

If you have any programs such as these, we would welcome you to send in a short description of the program, together with the program itself. You may send it to the attention of the LOCI User Application letter at Wang Laboratories in Tewksbury. We shall consider it a privilege to distribute the descriptions, and we feel that each user stands much to gain if there are ways of exchanging common programs. In all cases, we shall credit the author of the program whenever this is possible.

The attached first application is that for an Interpolation between data points. This is something which is often encountered in engineering. This first program as you may well guess, is developed inhouse at Wang Laboratories. If you would like the program itself, please enclose the bottom sheet of this Newsletter with your name and full address to us and we shall send it to you. While this program is a little bit on the complex side, we welcome any program which is either sophisticated or simple. In fact, we would welcome the simpler programs more since they would tend to be of more general interest.

Ned Chang  
Manager LOCI Division

LOCI User Application Letter # 1 — Interpolation (7-points)

A problem which often occurs in engineering analysis and statistics is to interpolate between known data points. For example, we may have 7 points,  $(X_0, Y_0), (X_1, Y_1), \dots, (X_7, Y_7)$ . Given these 7 points, we may wish to interpolate between them for non-observed values of  $X$ . This is usually done by means of linear interpolation between adjacent points. That is, we draw a straight line between the adjacent points, and interpolate on the line. This process is illustrated in the figure below:

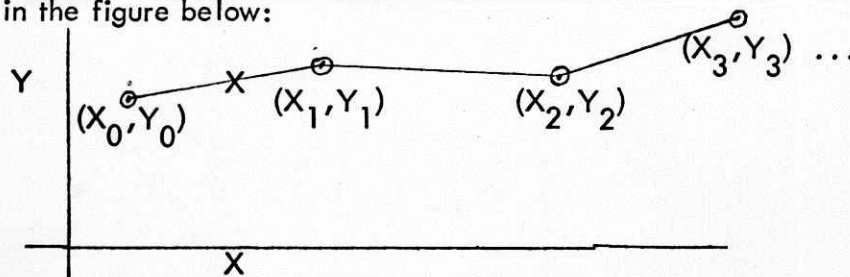


Figure 1

Another way in which we can do this is to draw a smooth curve through all of the points. We then interpolate by using the points on this curve. Drawing the curve can be done by visual inspection if we wish.

On the other hand, there is a simple classical technique for determining this curve. Technique is known as Lagrangian Interpolation. For 7 data points, the technique generates a  $6^{\circ}$  polynomial  $P(x)$  which will go through all 7 points. To interpolate for any point  $X$ , we simply evaluate the polynomial  $P(x)$ . The interpolation is generally excellent for all points that fall within the intervals between the known points. An illustration is once again given below:

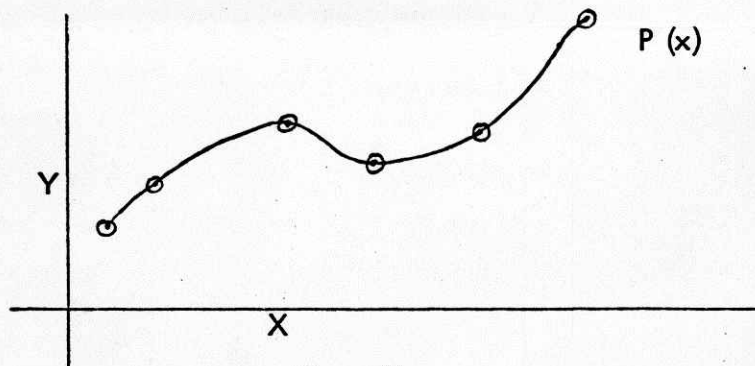


Figure 2

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## LOCI User Application Letter # 1 — Interpolation (7-points)

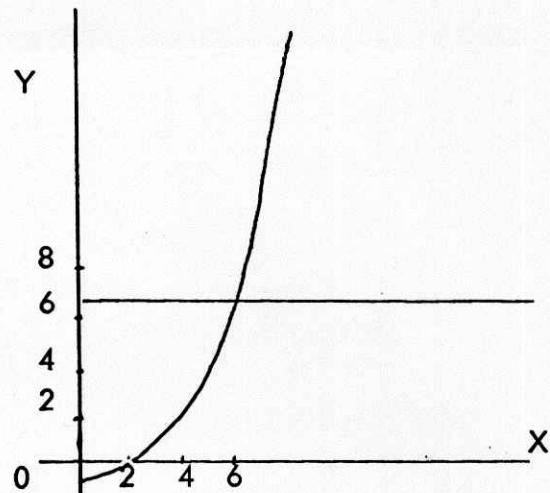
The LOCI program SI23 uses 7 points for  $6^{\circ}$  polynomial interpolation. Consider the following example, where we are given 7 points.

Given:

X	Y
1	-1.75
2	-1
4	2
5	4.25
6	7
10	23
11	28.25

Want:

X	Y
3	?
4.5	?
7	?
8	?
9	?



The first card in program SI23 is used to calculate the coefficients  $a_0, \dots, a_6$ . Once this is done, the second card is used to make any number of calculations, for example, to interpolate at the points  $X = 3, 4.5, 7, 8$  and  $9$ .

Operation Instructions

Card # 1. To operate the first card, key in a Y value, follow it with the corresponding X value, then follow that with all of the other X - values. Do this for all seven of the Y's. The coefficient values  $a_0 - a_6$  are automatically stored properly.

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LOCI User Application Letter # 1 — Interpolation (7-points)

1. AUTO DISP down, PRIME , card in reader
2. 1 . 7 5 ± Po (Yo)
3. 1 RUN (Xo)
4. 2 RUN (X1)
5. 4 RUN (X2)
6. 5 RUN (X3)
7. 6 RUN (X4)
8. 1 0 RUN (X5)
9. 1 1 RUN (X6)
10. 1 ± RUN (Y1)
11. 2 RUN (X1)
12. 1 RUN (X0)
13. 4 RUN (X2)
14. 5 RUN (X3)
15. 6 RUN (X4)
16. 1 0 RUN (X5)
17. 1 1 RUN (X6)
18. 2 RUN (Y2)
19. 4 RUN (X2)
20. 1 RUN (X0)
21. 1 RUN (X1)
22. 5 RUN (X3)
23. 6 RUN (X4)
24. 1 0 RUN (X5)
25. 1 1 RUN (X6)

$$\text{for } a_0 = \frac{Y_0}{(X_0 - X_1)(X_0 - X_2) \dots (X_0 - X_6)}$$

(-.00032407)

$$\text{for } a_1 = \frac{Y_1}{(X_1 - X_0)(X_1 - X_2) \dots (X_1 - X_6)}$$

$$\text{for } a_2 = \frac{Y_2}{(X_2 - X_0)(X_2 - X_1)(X_2 - X_3) \dots (X_2 - X_6)}$$

Repeat for all Y's.

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Card # 2 The second card performs the interpolation. For our example:

1. PRIME 3 Po (Interpolate for X = 3.)
2. Read answer: .2499999266
3. PRIME 4 · 5 Po (Interpolate for X = 4.5)
4. Read answer: 3.062499965
5. PRIME 7 Po (Interpolate for X = 7)
6. Read answer: 10.24999986
7. PRIME 8 Po (Interpolate for X = 8)
8. Read answer: 14.00000063
9. PRIME 9 Po (Interpolate for X = 9)
10. Read answer: 18.24999921

Notes: The known data where, in fact, points on the curve

$$Y = 1/4 X^2 - 2.$$

The quality of the above interpolations is evident.

Note that because of the programming procedure used, the points corresponding to  $X_0, X_1, X_2, \dots, X_6$  cannot be used.