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LOCI User Application #4

The Application Newsletter once again contains several interesting contributions.

L52-67 The last Newsletter contained a program by Dr. Fonda-Bonardi, for the solution of a Second Order Differential Equation. In this issue, he adds a remark, and a modification to the program. The solution is now general.

L53-67 Standard Deviation and Mean, Grouped Data. Mr. Douglas Lehman of the U.S. Post Office Department wrote this program, which accepts grouped data.

L50-67 Chi-Square Test. This program, also by Mr. Lehman, computes the X^2 variable.

L51-67 Chi-Square Distribution. This program was written at Wang Laboratories, Inc., to take advantage of our inherent log and exponential capability. The program essentially eliminates the need for searching the X^2 distribution table in a "Goodness-of-fit" test.

If you are interested in any of these applications, please fill out the form below and return it. In the meanwhile, we encourage you to send us your own applications.

Sincerely,

Ned Chang

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Manager, LOCI Division

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General Solution, 2nd Order Differential Equation

by

Dr. G. Fonda-Bonardi
Litton Systems

Regarding my last submitted program for the computation of second order differential equations, I would like to add a remark to the discussion: the general solution of a second order differential equation is

$$Y(x) = A\ Y_1(x) + B\ Y_2(x)$$

where A and B are arbitrary constants, and $Y_1(x)$ and $Y_2(x)$ are two distinct functions (e.g. sin x and cos x, or J(x) and N(x), etc.). The program computes the general y (x), in which the constants A and B are implicitly defined by the initial conditions. Thus, if the initial conditions are chosen to match a particular set of boundary values, the computed function is that mixture of $Y_1(x)$ and $Y_2(x)$ that matches the chosen boundary values, i.e the correct eigenfunction. To obtain a pure function, where either A or B is zero, one must choose the appropriate initial values. This was done for the examples submitted.

In writing the discussion of the program I felt that the handling of the case with the transcendental function was not satisfactory, because of the recommended programming was vague with the end left dangling. Accordingly, I rearranged the program to take care of this case, and I ended up with a program for computing systems of coupled second order differential equations:

$$y'' = F(x, y, y', z, z')$$

$$z'' = G(x, z, z, y, y', y'')$$

where F and G are any combination of operations on the variables that can be handled by the LOCI.

Equations of this kind occur in three cases:

1. Problems in physics or engineering that produce coupled equations, as for example, the motion of an electron in varying electric and magnetic fields, several problems in vibration analysis, etc.

2. Problems that produce a single second order differential equation with complex variables, as for example the propagation of electromagnetic fields in a slightly lossy plasma. The single differential equation splits in two when real and imaginary parts are separated,

3. Problems in which the functional relation F contains transcendental functions that cannot be computed algebraically (this is the case mentioned before).

I submit, therefore, a program that can handle such cases. It has the limitation that, since two functions are coded in Card 2, they cannot be as long to code as one could be in the previous program: however, card space is adequate for most practical cases. The commands for printing the results of the computation have been transferred to Card 2, so that the user can call for those variables that are of interest to him out of the many used during computation. This provides greater flexibility in setting up the problem but limits somewhat the card space available to the functions. Similarly, the number of iterations between printouts is coded in Card 2, at the end of the printout commands. This program runs slower than the previous one because it must handle almost twice as many operations per iteration.

I have tested out this program with a simple sine - cosine routine:

$$y'' = -z$$

$$z'' = -y$$

$$x_o = y_o = z_o = 0, \; y'_o = z'_o = 1.$$

Enclosed are the coding sheets for this, calling for printout of x, y, z, y' and z'. Comparison with a table of sines and cosines shows that the accuracy is quite good.

Operating Instructions

1. Put cards in Readers, AUTO DISP down.
2. Enter all initial values according to the table below:

S/MS	0	4	8	12
S0		Yo	Zo	
S1		ΔX	ΔX	
S2		Y'o	Z'o	
S3	Xo	Y''o	Z''o	

Note: ΔX should normally be set to .01, as discussed in the previous Newsletter, and the output printing or display interval is at .1 increments of X as programmed in the example.

3. Prime, Po

Equipment needed: LOCI-2abf, LOCI-2abh, LOCI-2abc, or LOCI-2ab. If no printer is available, put in Stop commands instead of WRITE commands for display.

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Standard Deviation
(Grouped Data)
by Douglas Lehman
Post Office Department

This standard deviation program was written by Mr. Lehman for use with grouped data. The formula is given below:

$$S = \left[\frac{\Sigma\, fiXi^2}{N} - \left(\frac{\Sigma\, fiXi}{N}\right)^2\right]^{1/2}$$

After calculating the standard deviation, the mean, $\overline{X}$, may be found in S1, and N is in S0.

Operating Procedure:

1. Put card in reader, AUTO DISP up, "AUTO".
2. Po
3. Enter Xi, RUN
4. Enter Fi, RUN
5. Pl for S.

Example:

X	F
1	2
2	3
3	0
4	2
5	1
6	0
7	1

Results S = 1.911627853

$\overline{X}$ = 3.111111109

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Chi-Square Test

by Douglas Lehman
Post Office Department
Research & Engineering

The Chi-Square (X^2) test is often used to compare an observed distribution with an assumed distribution. For example, in rolling a die 60 times, we would expect that each side of the die would come up 10 times. Any actual experiment would naturally not correspond exactly. We may test the fairness of the die by using the X^2 statistic. Mr. Lehman wrote a handy program to calculate X^2.

The formula for the statistic is simply that of a weighted sum of squares:

$$X^2 = \Sigma \frac{(Oi - Ei)^2}{Ei}$$

Oi = observed frequency

Ei = expected frequency

We observed that the larger the deviations, the larger is the variable X^2. (Having found the X^2 variable, we traditionally reach for the X^2 - Distribution table to make the test. However, LOCI-2 users may simply make use of the companion program L51-67.)

Operating Instructions

1. Card in Reader, AUTO DISP up, "AUTO"
2. Prime, Po
3. Index Oi, RUN Repeat for all pairs Oi and Ei.
   Index Ei, RUN (note that Ei must not be zero.)
4. P3, read X^2.
5. S3 ÷ W, Read N.

Example:

Suppose that in rolling the die, we obtained:

Face:	1	2	3	4	5	6
Frequency:	14	9	7	13	6	11

Our program yields,

$$X^2 = 5.2 \ (5.199999998)$$

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L51-67
5/18/67

Chi-Square Distribution

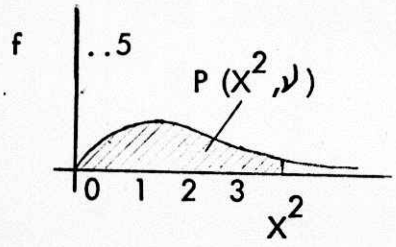
The Chi-Square (X^2) Distribution is a continuous distribution, used primarily to check the "Goodness-of-fit" of an assumed distribution when compared with actual observed frequencies.

$$f\,(X^2) = \frac{(X^2)^{(\nu/2-1)}\, e^{-X^2/2}}{(\nu/2 - 1)\,!\; 2^{\nu/2}}$$

$$X^2 = \Sigma\, \frac{(Oi - Ei)^2}{Ei}$$

ν = Degree of freedom

For $\nu = 4$, the distribution has the following form:



The probability integral may be approximated by

$$P\,(X^2,\nu) = \left(\frac{X^2}{2}\right)^{\nu/2} e^{-X^2/2}\; \frac{1}{(\frac{\nu}{2})!}\; S\,(X^2,\nu)$$

$$S\,(X^2,\nu) = 1 + \sum_{r=1}^{\infty} \frac{(X^2)^r}{(\nu+2)(\nu+4)\ldots(\nu+2r)}$$

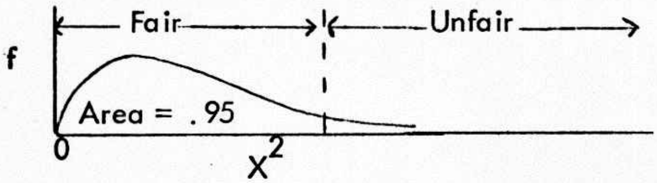
To eliminate the need for using the table, the program L51-67 calculates P (X^2, ν) by the above. Precision is better than 5-decimal places. This can be improved by using a more accurate value of TT.

Operating Instructions

1. Card In Reader, AUTO DISP up, "AUTO"
2. Index X^2, W—S1
3. Index ν, W—S2
4. Prime, Po
5. When machine stops,
   if is odd, 1 2 W—PC
   if is even, 2 6 W—PC
6. Read probability integral.

Chi-Square Distribution

Example 1: In the die-rolling experiment discussed in L50-67, we found that $X^2 = 5.2$. Suppose we wish to test the hypothesis that the die is fair, (i.e. each face is expected to show 10 times). Assume that we use a critical region of .05. In other words, we say the die is unfair if the probability that $X^2 \geq 5.2$ is .05 or less. This is the same as saying that, for the die to be fair, the probability that $X^2 < 5.2$ must be .95 or more.



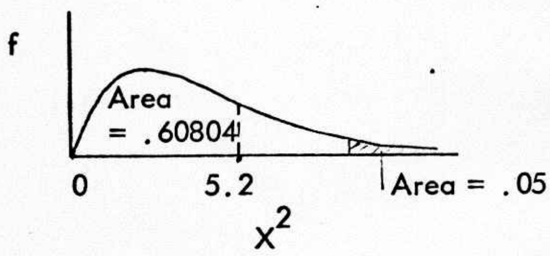
There are 5 degrees of freedom in the experiment. (Given the frequencies of any 5 faces of the die, we can find the frequency of the 6th face.)

To use the LOCI program:

1. Card In reader, AUTO DISP up, AUTO
2. 5 · 2 W—S1
3. 5 W—S2
4. Prime, Po
5. 1 2 RUN
6. Read,

   P (5.2, 5) = .60804

Therefore, the die is fair, since the X^2 probability of less than 5.2 is .60804, and a much larger value of X^2 would be needed to obtain .95.



Remarks on the Program

The program was written based upon the expansions:

ν odd: $$P(X^2,\nu) = \left(\frac{X^2}{2}\right)^{(\nu+1)/2} \frac{(4)^{(\nu+1)/2}}{2\,.\,6\,.\,10\ldots(2\nu)} \left(\frac{2}{X^2\,TT}\right)^{1/2} e^{-X^2/2}\Sigma'$$

ν even: $$P(X^2,\nu) = \left(\frac{X^2}{2}\right)^{(\nu+1)/2} \frac{(2)^{(\nu+1)/2}}{2\,.\,4\,.\,6\ldots\nu}\, e^{-X^2/2}\,\Sigma;$$

and, $$\Sigma = 1 + \frac{X^2}{\nu+2} + \frac{X^4}{(\nu+2)(\nu+4)} + \ldots + \frac{X^{2r}}{(\nu+2)\ldots(\nu+2r)} + \ldots$$

As written, the program contains an automatic convergence test on the last term in the series Σ. When the term becomes less than about 10^{-6}, the iteration stops and the answer is displayed. Thus, for even degrees of freedom, precision of the result is 10^{-6}. For odd degrees of freedom, the limit on precision is determined by the constant TT. Due to space limitations, TT was taken to be 3.1416 instead of 3.141592654. This restricts the precision to somewhere between 10^{-6} and 10^{-5}.

Note that in re-running the program it is not necessary to re-enter X^2 in S1. But it is necessary to re-enter ν in S2 before giving the commands Prime, Po.

Overflow may occur for overly large values of X^2. This may happen in two cases. First, if $X^2/2 > 40$, an error may result at step # 7 in computing $e^{-x^2}/2$. Second an error condition may occur if any single term in the series Σ exceed 10^{-9}. There is also a third possbility that the log of the coefficient to the series (log K) exceed +40. The first and third cases can be handled if two card readers are used. All these cases normally imply that $P(X^2,\nu)$ is almost 1, and that the critical region is almost 0.