WANG
LABORATORIES, INC. 836 NORTH ST., TEWKSBURY, MASS. 01876, TEL. (617) 851-7311

June 1, 1967

LOCI User Application Letter #4

Enclosed please find your programs for LOCI User Application Letter #4. The enclosures are:

L52-67	General Solution, Second Order Differential Equation.
L53-67	Standard Deviation (Gouped Data)
L50-67	Chi-Square Test
L51-67	Chi-Square Distribution

Please remember that if you have any applications of general interest, we shall be happy to distribute them.

Sincerely,

WANG LABORATORIES, INC.

Ned Chang B.W.

Ned Chang
Manager, LOCI Division

NC/bw

WANG LABORATORIES, INC. 836 NORTH ST., TEWKSBURY, MASS. 01876, TEL. (617) 851-7311

Calculating & Computing Calculations

General Solution, 2nd Order Differential Equation

by
Dr. G. Fonda-Bonardi
Litton Systems

Regarding my last submitted program for the computation of second order differential equations, I would like to add a remark to the discussion: the general solution of a second order differential equation is

$$Y(x) = A\ Y_1(x) + B\ Y_2(x)$$

where A and B are arbitrary constants, and $Y_1(x)$ and $Y_2(x)$ are two distinct functions (e.g. sin x and cos x, or J(x) and N(x), etc.). The program computes the general y (x), in which the constants A and B are implicitly defined by the initial conditions. Thus, if the initial conditions are chosen to match a particular set of boundary values, the computed function is that mixture of $Y_1(x)$ and $Y_2(x)$ that matches the chosen boundary values, i.e. the correct eigenfunction. To obtain a pure function, where either A or B is zero, one must choose the appropriate initial values. This was done for the examples submitted.

In writing the discussion of the program I felt that the handling of the case with the transcendental function was not satisfactory, because of the recommended programming was vague with the end left dangling. Accordingly, I rearranged the program to take care of this case, and I ended up with a program for computing systems of coupled second order differential equations:

$$y'' = F(x, y, y', z, z')$$

$$z'' = G(x, z, z, y, y', y'')$$

where F and G are any combination of operations on the variables that can be handled by the LOCI.

Equations of this kind occur in three cases:

1. Problems in physics or engineering that produce coupled equations, as for example, the motion of an electron in varying electric and magnetic fields, several problems in vibration analysis, etc.

2. Problems that produce a single second order differential equation with complex variables, as for example the propagation of electromagnetic fields in a slightly lossy plasma. The single differential equation splits in two when real and imaginary parts are separated,

3. Problems in which the functional relation F contains transcendental functions that cannot be computed algebraically (this is the case mentioned before).

I submit, therefore, a program that can handle such cases. It has the limitation that, since two functions are coded in Card 2, they cannot be as long to code as one could be in the previous program; however, card space is adequate for most practical cases. The commands for printing the results of the computation have been transferred to Card 2, so that the user

General Solution, 2nd Order Differential Equation

can call for those variables that are of interest to him out of the many used during computation. This provides greater flexibility in setting up the problem but limits somewhat the card space available to the functions. Similarly, the number of iterations between printouts is coded in Card 2, at the end of the printout commands. This program runs slower than the previous one because it must handle almost twice as many operations per iteration.

I have tested out this program with a simple sine - cosine routine:

$$y'' = -z$$

$$z'' = -y$$

$$x_o = y_o = z_o = 0, \; y'_o = z'_o = 1.$$

Enclosed are the coding sheets for this, calling for printout of x, y, z, y' and z'. Comparison with a table of sines and cosines shows that the accuracy is quite good.

Operating Instructions

1. Put cards in Readers, AUTO DISP down.
2. Enter all initial values according to the table below:

S/MS	0	4	8	12
S0		Yo	Zo	
S1		ΔX	ΔX	
S2		Y'o	Z'o	
S3	Xo	Y"o	Z"o	

Note: ΔX should normally be set to .01, as discussed in the previous Newsletter, and the output printing or display interval is at .1 increments of X as programmed in the example.

3. Prime, Po

Equipment needed: LOCI-2abf, LOCI-2abh, LOCI-2abc, or LOCI-2ab. If no printer is available, put in Stop commands instead of WRITE commands for display.

LOCI PROGRAM

Card # 1   No. L52 - 67   Date: May 15, 1967

No.	Cmd	Code	Comment	No.	Cmd	Code	Comment
00	M			40	S3--W	57	
01	N			41	−	15	
02	XPC	41		42	X	12	
03	5	25		43	S1--W	53	
04	3	23		44	X	12	
05	Store	64		45	2	22	
06	MS	10		46	÷	17	
07	5	25		47	Ln-1	14	
08	3	23		48	W--A	44	
09	Store	64		49	S2--W	55	
10	W--A	44		50	+	13	
11	CLW	02		51	W--S2	54	
12	S3--W	57		52	Restore	65	
13	+	13		53	S1--W	53	
14	W--S3	56		54	S0--A	51	
15	0	20		55	X	12	
16	±	33		56	S2--W	55	
17	Store	64		57	X	12	
18	4	24		58	Ln-1	14	
19	Store	64		59	+	13	
20	Q			60	S1--W	53	
21	±	33		61	□	06	
22	Store	64		62	S3--W	57	
23	4	24		63	X	12	
24	Store	64		64	2	22	
25	CLW	02		65	÷	17	
26	±	33		66	Ln-1	14	
27	Store	64		67	+	13	
28	W--S3	56		68	A--S0	50	
29	Q			69	S1--W	53	
30	±	33		70	X	12	
31	Store	64		71	S3--W	57	
32	W--S3	56		72	X	12	
33	DECDC	66		73	Ln-1	14	
34	TestDC	70		74	W--A	44	
35	CLW	02		75	S2--W	55	
36	MS	10		76	+	13	
37	P1	61		77	W--S2	54	
38	CLW	02		78	S1--W	53	
39	P0	60		79	Restore	65	

S \ MS	0	4	8	12
S0		Yo	Zo	
S1		ΔX	ΔX	
S2		Y'o	Z'o	
S3	Xo	Y"o	Z"o	

Coupled second order differential equations:

$y'' = F(X, y, y', z, z')$

$z'' = G(x, z, z', y, y', y'')$

Card 1

Code MN (step 00,01) for start of printout commands.

Code Q (steps 20 and 29) for start of function G.

Enter all initial values in memories: $x_o, y_o, y'_o, y''_o, z_o, z'_o, z''_o$

Enter Δx = 0.001 in MS 4 S1 and MS 8 S1.

Start with PRIME P0

Dr. Bonardi

List of operations.

00	Not assigned	40	W→PC
01	Clear error	41	W→XPC
02	Clear W	42	W→DC
03	Clear A	43	DC→W
04	√‾	44	W→A
05	1/√‾	45	A→W
06	□	46	W→L
07	1/□	47	L→W
10	Step MSC	50	$A \to S_0$
11	Write	51	$S_0 \to A$
12	×	52	$W \to S_1$
13	+	53	$S_1 \to W$
14	LN-1	54	$(W \to S_2)$
15	−	55	$(S_2 \to W)$
16	·	56	$(W \to S_3)$
17	÷	57	$(S_3 \to W)$
20	0	60	P_0 (Set PC to 00)
21	1	61	P_1 (" " " 03)
22	2	62	P_2 (" " " 06)
23	3	63	P_3 (" " " 09)
24	4	64	Store PC, DC then W→PC
25	5	65	Recall PC, DC
26	6	66	Decrement DC
27	7	67	Test error
30	8	70	Test DC = 0
31	9	71	Test A = 0
32	Run	72	Reserved
33	±	73	Test W for — sign
34	Input MX	74	Test L for — exponent
35	Output MX	75	Carriage return
36	Prime	76	Read
37	Stop	77	Not assigned

*Commands 10, 11, 34, 35, 75, and 76 are available only when special options are purchased.

LOCI PROGRAM   Card # 2   No. L52 - 67   Date: May 15, 1967

No.	Cmd	Code	Comment	No.	Cmd	Code	Comment
00	MS	10		40			
01	MS	10		41			
02	S0--A	51		42			
03	CLW	02		43			
04	MS	10		44			
05	A--W	45		45			
06	±	33		46			
07	W--A	44		47			
08	Restore	65		48			
09				49			
10				50			
11				51			
12				52			
13				53			
14				54			
15				55			
16				56	S3--W	57	MN = 56
17				57	Write	11	
18				58	MS	10	
19				59	S0--A	51	
20				60	A--W	45	
21				61	Write	11	
22				62	MS	10	
23				63	S0--A	51	
24				64	A--W	45	
25				65	Write	11	
26				66	CLW	02	
27				67	MS	10	
28				68	S2--W	55	
29				69	Write	11	
30	S0--A	51	Q = 3	70	MS	10	
31	A--W	45		71	S2--W	55	
32	±	33		72	Write	11	
33	W--A	44		73	CLW	02	
34	MS	10		74	MS	10	
35	Restore	65		75	1	21	
36				76	0	20	
37				77	W--DC	42	
38				78	P1	61	
39				79			

S \ MS	0	4	8	12
S0		y	z	
S1		Δx	Δx	
S2		y'	z'	
S3	X	y"	z"	

Coupled Second Order Differential Equations: $y'' = F(x, y, y', z, z')$
Card 2 $z'' = G(x, z, z', y, y', y'')$

Start coding function F at step -00. Card is entered with MS 0 and must be left with MS 4 and y" in A and W. End with RESTORE (65).

Start coding function G at step -00. Card is entered with MS 4 and must be left with MS 8 and z" in A and W. End with RESTORE (65).

Start coding printout of desired variables at step -MN. Card is entered with MS 0 and must be left with MS 4. End with sequence 1 0 W--DC P1 (21 20 42 61), change 1 0 if other number of iterations is desired.

Initial conditions: $x_o = y_o = z_o = y''_o = z''_o = 0$ $y' = z' = 1$

Dr. Bonardi

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Standard Deviation
(Grouped Data)

by Douglas Lehman
Post Office Department

This standard deviation program was written by Mr. Lehman for use with grouped data. The formula is given below:

$$S = \left[\frac{\Sigma\, fiXi^2}{N} - \left(\frac{\Sigma\, fiXi}{N}\right)^2 \right]^{1/2}$$

After calculating the standard deviation, the mean, $\bar{X}$, may be found in S1, and N is in S0.

Operating Procedure:

1. Put card in reader, AUTO DISP up, "AUTO".
2. Po
3. Enter Xi, RUN
4. Enter Fi, RUN
5. P1 for S.

Example:

X	F
1	2
2	3
3	0
4	2
5	1
6	0
7	1

Results $S = 1.911627853$

$\bar{X} = 3.111111109$

LOCI PROGRAM Standard Deviation No. L53 - 67 Date: May 16, 1967

No.	Cmd	Code	Comment	No.	Cmd	Code	Comment
00	0	20		40	S0--A	51	
01	6	26		41	A--W	45	
02	W--PC	40		42	÷	17	
03	3	23		43	Ln-1	14	$\frac{\Sigma F X}{\Sigma F}$
04	8	30		44	W--S1	52	Store $\frac{\Sigma FX}{N}$ in S1
05	W--PC	40		45	S3--W	57	
06	Prime	36		46	X	12	
07	A--S0	50		47	S0--A	51	
08	W--S1	52		48	A--W	45	
09	W--S2	54		49	÷	17	
10	W--S3	56		50	Ln-1	14	$(\frac{\Sigma + X}{N})^2$
11	Stop	37	Enter X	51	W--A	44	
12	W--S1	52	Store X in S1	52	S1--W	53	
13	Stop	37	Enter F	53		06	
14	S0--A	51		54	Ln-1	14	$(\frac{\Sigma + X}{N})^2$
15	+	13	Σ F	55	−	15	
16	A--S0	50	Store ΣF in S0	56	A--W	45	
17	X	12		57	√	04	
18	S1--W	53		58	:m-1	14	S
19	X	12		59	Stop	37	
20	S2--W	55		60			
21	W--A	44		61			
22	Ln-1	14	F X	62			
23	+	13	Σ F X	63			
24	X	12		64			
25	A--W	45		65			
26	W--S2	54	Store ΣF X in S2	66			
27	S1--W	53		67			
28	X	12		68			
29	Ln-1	14	F X^2	69			
30	W--A	44		70			
31	S3--W	57		71			
32	+	13	ΣF X^2	72			
33	A--W	45		73			
34	W--S3	56	Store ΣF X^2 in S3	74			
35	1	21		75			
36	1	21		76			
37	W--PC	40		77			
38	S2--W	55		78			
39	X	12		79			

$$S = \left\{ \frac{\Sigma Fi\, Xi^2}{N} - \left(\frac{\Sigma Fi\, Xi}{N}\right)^2 \right\}^{1/2}$$

Note: Auto Disp In Up Position

Push Po

Enter X1
Push RUN

Enter F1
Push RUN

Repeat for Xi, Fi

Push P1 for S

S0 = Σ Fi = N

S1 = $\overline{Xi}$

S2 = Σ Fi Xi

S3 = Σ Fi Xi^2

List of operations.

00	Not assigned	40	W→PC
01	Clear error	41	W→XPC
02	Clear W	42	W→DC
03	Clear A	43	DC→W
04	√	44	W→A
05	1/√	45	A→W
06	□	46	W→L
07	1/□	47	L→W
10	Step MSC	50	A→S_0
11	Write	51	S_0→A
12	×	52	W→S_1
13	+	53	S_1→W
14	LN-1	54	(W→S_2)
15	−	55	(S_2→W)
16	·	56	(W→S_3)
17	÷	57	(S_3→W)
20	0	60	P_0 (Set PC to 00)
21	1	61	P_1 (" " " 03)
22	2	62	P_2 (" " " 06)
23	3	63	P_3 (" " " 09)
24	4	64	Store PC, DC then W→PC
25	5	65	Recall PC, DC
26	6	66	Decrement DC
27	7	67	Test error
30	8	70	Test DC=0
31	9	71	Test A=0
32	Run	72	Reserved
33	±	73	Test W for — sign
34	Input MX	74	Test L for — exponent
35	Output MX	75	Carriage return
36	Prime	76	Read
37	Stop	77	Not assigned

*Commands 10, 11, 34, 35, 75, and 76 are available only when special options are purchased.

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L50-67
5/18/67

Chi-Square Test

by Douglas Lehman
Post Office Department
Research & Engineering

The Chi-Square (X^2) test is often used to compare an observed distribution with an assumed distribution. For example, in rolling a die 60 times, we would expect that each side of the die would come up 10 times. Any actual experiment would naturally not correspond exactly. We may test the fairness of the die by using the X^2 statistic. Mr. Lehman wrote a handy program to calculate X^2.

The formula for the statistic is simply that of a weighted sum of squares:

$$X^2 = \Sigma \frac{(Oi - Ei)^2}{Ei}$$

Oi = observed frequency

Ei = expected frequency

We observed that the larger the deviations, the larger is the variable X^2. (Having found the X^2 variable, we traditionally reach for the X^2 - Distribution table to make the test. However, LOCI-2 users may simply make use of the companion program L51-67.)

Operating Instructions

1. Card in Reader, AUTO DISP up, "AUTO"
2. Prime, Po
3. Index Oi, RUN Repeat for all pairs Oi and Ei.
   Index Ei, RUN (note that Ei must not be zero.)
4. P3, read X^2.
5. S3→W, Read N.

Example:

Suppose that in rolling the die, we obtained:

Face:	1	2	3	4	5	6
Frequency:	14	9	7	13	6	11

Our program yields,

$X^2 = 5.2$ (5.199999998)

LOCI PROGRAM CHI-Square Test

No. L50-67 Date: 5/15/67

No.	Cmd	Code	Comment	No.	Cmd	Code	Comment
00	A--S0	50		40	W--S3	56	
01	W--S1	52		41	0	20	
02	W--S2	54		42	4	24	
03	W--S3	56		43	W--PC	40	
04	Stop	37		44			
05	W--A	44		45			
06	CLR W	02		46			
07	2	22		47			
08	W--PC	40		48			
09	S2--W	55		49			
10	Stop	37		50			
11				51			
12				52			
13				53			
14				54			
15				55			
16				56			
17				57			
18				58			
19				59			
20	Stop	37		60			
21	W--S1	52		61			
22	—	15		62			
23	A--W	45		63			
24	□	06		64			
25	Ln-1	14		65			
26	X	12		66			
27	S1--W	53		67			
28	÷	17		68			
29	Ln-1	14		69			
30	W--A	44		70			
31	S2--W	55		71			
32	+	13		72			
33	A--W	45		73			
34	W--S2	54		74			
35	S3--W	57		75			
36	W--A	44		76			
37	1	21		77			
38	+	13		78			
39	A--W	45		79			

$$X^2 = \sum_{i=1}^{k} \frac{(Oi - Ei)^2}{E^i}$$

Push Prime
Po
Enter O
Push Run
Enter E
Push Run
Repeat for Oi, Ei
Push P3 for CHI-Square
Push S3 for N

Note: AUTO Display in up position.

by Douglas Lehman
Post Office Department
Research & Engineering

List of operations.

00	Not assigned	40	W→PC
01	Clear error	41	W→XPC
02	Clear W	42	W→DC
03	Clear A	43	DC→W
04	√	44	W→A
05	1/√	45	A→W
06	□	46	W→L
07	1/□	47	L→W
10	Step MSC	50	$A \to S_0$
11	Write	51	$S_0 \to A$
12	×	52	$W \to S_1$
13	+	53	$S_1 \to W$
14	LN-1	54	$(W \to S_2)$
15	−	55	$(S_2 \to W)$
16	·	56	$(W \to S_3)$
17	÷	57	$(S_3 \to W)$
20	0	60	P_0 (Set PC to 00)
21	1	61	P_1 (" " " 03)
22	2	62	P_2 (" " " 06)
23	3	63	P_3 (" " " 09)
24	4	64	Store PC, DC then W→PC
25	5	65	Recall PC, DC
26	6	66	Decrement DC
27	7	67	Test error
30	8	70	Test DC = 0
31	9	71	Test A = 0
32	Run	72	Reserved
33	±	73	Test W for — sign
34	Input MX	74	Test L for — exponent
35	Output MX	75	Carriage return
36	Prime	76	Read
37	Stop	77	Not assigned

*Commands 10, 11, 34, 35, 75, and 76 are available only when special options are purchased.

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L51-67
5/18/67

Chi-Square Distribution

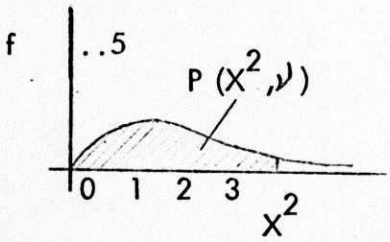
The Chi-Square (X^2) Distribution is a continuous distribution, used primarily to check the "Goodness-of-fit" of an assumed distribution when compared with actual observed frequencies.

$$f(X^2) = \frac{(X^2)^{(\nu/2-1)}\, e^{-X^2/2}}{(\nu/2-1)!\, 2^{\nu/2}}$$

$$X^2 = \Sigma \frac{(Oi - Ei)^2}{Ei}$$

ν = Degree of freedom

For $\nu = 4$, the distribution has the following form:



The probability integral may be approximated by

$$P(X^2,\nu) = \left(\frac{X^2}{2}\right)^{\nu/2} e^{-X^2/2} \frac{1}{(\frac{\nu}{2})!} S(X^2,\nu)$$

$$S(X^2,\nu) = 1 + \sum_{r=1}^{\infty} \frac{(X^2)^r}{(\nu+2)(\nu+4)\ldots(\nu+2r)}$$

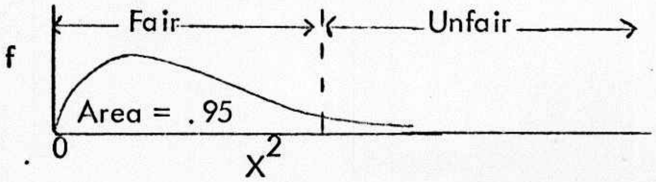
To eliminate the need for using the table, the program L51-67 calculates P (X^2, ν) by the above. Precision is better than 5-decimal places. This can be improved by using a more accurate value of TT.

Operating Instructions

1. Card In Reader, AUTO DISP up, "AUTO"
2. Index X^2, W—S1
3. Index ν, W—S2
4. Prime, Po
5. When machine stops,
   if is odd, 1 2 W—PC
   if is even, 2 6 W—PC
6. Read probability integral.

Chi-Square Distribution

Example 1: In the die-rolling experiment discussed in L50-67, we found that $X^2 = 5.2$. Suppose we wish to test the hypothesis that the die is fair, (i.e. each face is expected to show 10 times). Assume that we use a critical region of .05. In other words, we say the die is unfair if the probability that $X^2 \geq 5.2$ is .05 or less. This is the same as saying that, for the die to be fair, the probability that $X^2 < 5.2$ must be .95 or more.



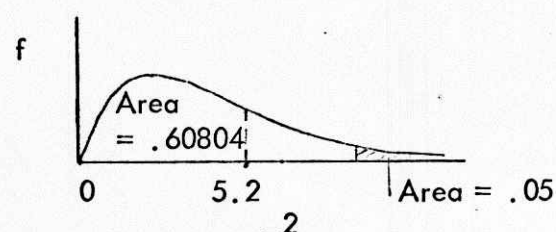
There are 5 degrees of freedom in the experiment. (Given the frequencies of any 5 faces of the die, we can find the frequency of the 6th face.)

To use the LOCI program:

1. Card In reader, AUTO DISP up, AUTO
2. 5 · 2 W—S1
3. 5 W—S2
4. Prime, Po
5. 1 2 RUN
6. Read,

   P (5.2, 5) = .60804

Therefore, the die is fair, since the X^2 probability of less than 5.2 is .60804, and a much larger value of X^2 would be needed to obtain .95.



Chi-Square Distribution

Remarks on the Program

The program was written based upon the expansions:

ν odd: $$P(X^2,\nu) = \left(\frac{X^2}{2}\right)^{(\nu+1)/2} \frac{(4)^{(\nu+1)/2}}{2 \cdot 6 \cdot 10 \ldots (2\nu)} \left(\frac{2}{X^2 \pi}\right)^{1/2} e^{-X^2/2} \Sigma'$$

ν even: $$P(X^2,\nu) = \left(\frac{X^2}{2}\right)^{(\nu+1)/2} \frac{(2)^{(\nu+1)/2}}{2 \cdot 4 \cdot 6 \ldots \nu} e^{-X^2/2} \Sigma;$$

and, $$\Sigma = 1 + \frac{X^2}{\nu+2} + \frac{X^4}{(\nu+2)(\nu+4)} + \ldots + \frac{X^{2r}}{(\nu+2) \ldots (\nu+2r)} + \ldots$$

As written, the program contains an automatic convergence test on the last term in the series Σ. When the term becomes less than about 10^{-6}, the iteration stops and the answer is displayed. Thus, for even degrees of freedom, precision of the result is 10^{-6}. For odd degrees of freedom, the limit on precision is determined by the constant TT. Due to space limitations, TT was taken to be 3.1416 instead of 3.141592654. This restricts the precision to somewhere between 10^{-6} and 10^{-5}.

Note that in re-running the program it is not necessary to re-enter X^2 in S1. But it is necessary to re-enter ν in S2 before giving the commands Prime, Po.

Overflow may occur for overly large values of X^2. This may happen in two cases. First, if $X^2/2 > 40$, an error may result at step # 7 in computing $e^{-x^2/2}$. Second an error condition may occur if any single term in the series Σ exceed 10^{-9}. There is also a third possbility that the log of the coefficient to the series (log K) exceed +40. The first and third cases can be handled if two card readers are used. All these cases normally imply that $P(X^2,\nu)$ is almost 1, and that the critical region is almost 0.

Example 2: Statistical Investigation of order processing

Some of the applications published to date furnish us with tools to perform some interesting statistical analysis. We choose an example which makes use of four applications discussed in these Newsletters:

L53-67	Standard Deviation for Grouped Data,
L46-67	Poisson Distribution,
L50-67	X^2 - Statistic,
L51-67	X^2 - Distribution

Consider the following daily log for the order processing for certain products.

Table I

Date	K, Number of Orders
3-6	3
3-7	0
3-8	4
3-9	0
3-10	1
3-13	2
3-14	0
3-15	1
3-16	0
3-17	1
3-20	1
3-21	0
3-22	0
3-23	2
3-24	2
3-27	1
3-28	1
3-29	0
3-30	4
3-31	0

We next take the table and group it to obtain a new table. The new table consists of K, the numbers of orders per day, and N_k, the number of days with K orders.

k, Daily Number of Orders	N_k, Number of Days with k Orders
0	8
1	6
2	3
3	1
4	2
5	0
6	0
	20

$\Sigma k\, N_k = 23$

Estimate of the mean,

The total number of orders is 23, spread over 20 days. The average daily rate is $\lambda = 23/20 = 1.15$. (For a large table, these calculations may be performed using the group standard deviation program L53-67. The variables K and N_k correspond to the inputs Xi and Fi)

Theoretical Poisson Distribution $P(K,\lambda)$

Table 2 appears to conform to a Poisson Distribution. In order to see whether it does, we generate a table by using the Poisson Distribution program L46-67. Our expected value for K is $\lambda = 1.15$. Results are tabulated below. The second column consists of the probability frequencies, $P(K,\lambda)$. Since the distribution is over 20 days, the third column consists of the theoretical number of days $N_k' = 20\,P(K, 1.15)$ in which we have k orders.

Table 3 (Poisson Distribution - Theoretical) = 1.15

K, Daily Number of Orders	P(K, 1.15) Theoretical Probability for N_k	$N_k' = 20\,P(k,\lambda)$ Theoretical Number of days with K orders
0	.3166	6.33
1	.3641	7.83
2	.2094	4.19
3	.0803	1.61
4	.0231	0.46
5	.0053	0.11
6	.0010	0.02

X^2 - Test between N_k and N_k'

To see if our order rate behaves in accordance with the theoretical, Poisson Distribution, we can construct a new table.

Table 4 (Observed vs. Theoretical Distribution)

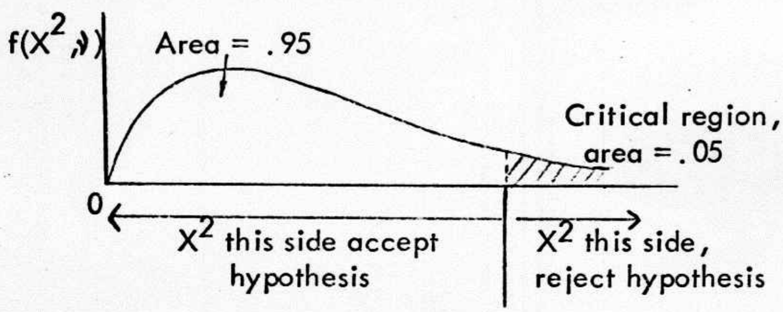
N_k	N_k'
8	6.33
6	7.83
3	4 19
1	1.61
2	0.46
0	0.11
0	0.02

A visual comparison shows that the two columns are quite close. We are led to conclude that the distribution of order rate follows a Poisson Distribution.

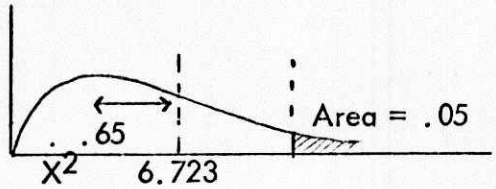
This hypothesis can be vigorously tested with the X^2-test (described in our Applications Newsletters # 3 and # 4, L50-67 and L51-67). The program L50-67 may be used to calculate the X^2-value; where the observed values are N_k and the theoretical values are N_k'. In this case, the data in Table 4 yield a value $X^2 = 6.723$.

To test the hypothesis, we choose a critical region of .05, and use the X^2-Distribution (L51-67).

In this example, there are 7 lines in the table, and hence there are 6 degrees of freedom. With a critical region (right tail) of .05, we are saying that if X^2 falls in this region, the hypothesis is rejected; or, the observed distribution does not match the theoretical.



On the other hand, if X^2 falls outside (to the left) of this region, the hypothesis is accepted. The program L51-67 gives P (X^2, ν), or the area under the curve from 0 to X^2 on the abscissa. Therefore, if P (6.723,6) is less than .95, the hypothesis can be accepted. We find that P(6.723,6) = .65. Thus, we can accept the hypothesis and say that the observations N_k conform to the Poisson Distribution.



Remarks

1. In the course of this discussion, several tables were constructed to facilitate the exposition. Naturally, it would be more convenient to construct a more compact table; such as the one below, combining Tables 2, 3, and 4.

Table 5

K, Daily Order Rate	N_k, observed days with rate K	P(K,1.15)Theoretical frequency for K		20 P(K,1.15)Theoretical number of days with rate K
0	8	.3166		6.33
1	6	.3641	89%	7.83
2	3	.2094		4.19
3	1	.0803		1.61
4	2	.0231		0.46
5	0	.0053		0.11
6	0	.0010		0.02

$N = 20, \lambda = 1.15, \quad \nu = 6$

$X^2 = 6.723, \quad P(6.723,6) = .65$

2. The third column can be interpreted to make utilitarian decisions. For order rates which follow a Poisson Distribution, we can see that if we have the capacity to process 2 orders a day, 89% of the time, we can do everything in one day. Suppose that it takes one man per order, with two men, we have adequate capacity 89% of the time, with 3 men, it becomes 97% of the time. While manufacturing and engineering decisions cannot be made this way due to other factors, suppose K is the failure rate of equipment. A tabulation such as Table 5 can be effectively used to establish service requirements.

3. In this example, we used a critical region of .05. Normal X^2 - distribution tables tabulate in increments of .05. However, the program L51-67 can be used with any sized critical region, such as .03, .12, etc.

4. Note that the X^2-test can be used to check the stability of the system. For example, suppose that next month, the X^2 value is so large that $P(X^2, \nu) > .95$. This means that the observed distribution no longer fits the theoretical and further investigation would be required.

LOCI PROGRAM X^2 Distribution

No. L51-67 Date: 5/15/67

No.	Cmd	Code	Comment	No.	Cmd	Code	Comment
00	S1--W	53	X^2	40	9	31	continue
01	X	12		41	W--PC	40	
02	2	22		42	L--W	47	
03	÷	17		43	W--S3	56	in S3
04	Ln-1	14	$X^2/2$	44	1	21	
05	W--S3	56	in S3	45	W--A	44	
06	±	33		46	A--S0	50	Σ0 = 1
07	W--L	46	$e^{-X^2/2}$	47	A--W	45	Last Term
08	S2--W	55	ν	48	X	12	
09	—	15	-ν in A	49	S2--W	55	
10	Stop	37		50	W--A	44	ν + 2r
11	W--PC	40	even or odd	51	2	22	
12	4	24	ν odd	52	+	13	new ν + 2
13	W--DC	42		53	A--W	45	
14	S2--W	55	ν	54	W--S2	54	
15	—	15	-2ν in A	55	÷	17	$\frac{Ti}{\nu + 2r}$
16	S3--W	57	$(X^2/2)^{-1/2}$	56	S1--W	53	
17	1/√	05		57	X	12	$X^2 \frac{Ti}{\nu+2r}$
18	3	23		58	Ln-1	14	new Ti=Ti+1
19	.	16		59	S0--A	51	Σ
20	1	21		60	+	13	
21	4	24		61	A--S0	50	new Σ
22	1	21		62	W--A	44	Ti
23	6	26		63	X	12	
24	1/√	05	$\sqrt{\frac{2}{\pi X^2}}\, e^{-X^2}$	64	9	31	
25	Test Error	67	skip 3 steps	65	9	31	
26	2	22	ν even	66	9	31	
27	W--DC	42		67	□	06	Ti 10^6
28				68	L--W	47	
29	S3--W	57	$X^2/2$	69	W--?	73	Ti $10^6 < 1$?
30	X	12		70	4	24	
31	A--W	45	-ν or -2ν	71	7	27	No
32	÷	17		72	W--PC	40	
33	DC--W	43	2 or 4	73	S3--W	57	Yes
34	?	13	new -ν	74	W--L	46	ln ( )
35	X	12		75	S0--A	51	Σ
36	A--W	45	new ν > 0 ?	76	A--W	45	
37	±	33		77	X	12	
38	W--?	73		78	Ln-1	14	( )Σ
39	2	22		79	Stop	37	

DC 4 or 2
S0 Σ
S1 X^2
S2 ν, ν + 2r
S3 $X^2/2$, log K

$$K = \left(\frac{X^2}{2}\right)^{\nu/2} e^{-X/2} \left(\frac{\nu}{2}\right)!$$

$$\Sigma = 1 + \sum_{r=1} \frac{(X^2)^r}{(\nu+2)(\nu+4)\dots(\nu+2r)}$$

$$Q(X^2, \nu) = K\Sigma$$

Note: Error $< 10^{-5}$

1. Key in X^2, W--S1
2. Key in ν, W--S2 (Degree of freedom)
3. Prime, Po
4. When machine stops,
   a) if ν is odd, (1,3,5,7...) 1 2 RUN.
   b) if ν is even (2,4,6,...) 2 6 RUN.
5. Read answer $P(X^2, \nu)$

List of operations.

00	Not assigned	40	W→PC
01	Clear error	41	W→XPC
02	Clear W	42	W→DC
03	Clear A	43	DC→W
04	√	44	W→A
05	1/√	45	A→W
06	□	46	W→L
07	1/□	47	L→W
10	Step MSC	50	$A \to S_0$
11	Write	51	$S_0 \to A$
12	×	52	$W \to S_1$
13	+	53	$S_1 \to W$
14	LN-1	54	$(W \to S_2)$
15	−	55	$(S_2 \to W)$
16	·	56	$(W \to S_3)$
17	÷	57	$(S_3 \to W)$
20	0	60	P_0 (Set PC to 00)
21	1	61	P_1 (" " " 03)
22	2	62	P_2 (" " " 06)
23	3	63	P_3 (" " " 09)
24	4	64	Store PC, DC then W→PC
25	5	65	Recall PC, DC
26	6	66	Decrement DC
27	7	67	Test error
30	8	70	Test DC = 0
31	9	71	Test A = 0
32	Run	72	Reserved
33	±	73	Test W for — sign
34	Input MX	74	Test L for — exponent
35	Output MX	75	Carriage return
36	Prime	76	Read
37	Stop	77	Not assigned

*Commands 10, 11, 34, 35, 75, and 76 are available only when special options are purchased.